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MICROWAVE BREAKDOWN CALCULATIONS
THAT INCLUDE THE EFFECTS OF PREIONIZATION
IN NEON AND NEON-ARGON MIXTURES

Mark R. Foster

Air Force Cambridge Research Laboratories L. G. Hanscom Field, Massachusetts

6 September 1973

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# Microwave Breakdown Calculations That Include the Effects of Preionization in Neon and Neon-Argon Mixtures

#### 1. INTRODUCTION

To calculate the electric field required to initiate microwave breakdown in a partially ionized gas, a computer program was written and an expression derived for the rate of electron loss due to diffusion in the free and ambipolar modes. The breakdown calculations reported here are based on a simplified form of this diffusion expression and therefore include the effects of charge separation resulting from preionization. Ionization by electron-metastable collisions and other mechanisms that may become important in dealing with preionization are not treated here.

#### 1.1 The Breakdown Criterion

Microwave breakdown occurs in a gas when the rate of electron production equals or infinites, mally exceeds the rate of electron loss. The electric field accelerates the electrons which subsequently, through collision with neutral gas atoms, randomize this ordered oscillatory energy. Since energy is coupled into the electron constituent, an electron avalanche will occur if a sufficient number of electrons accumulate a random kinclic energy greater than the ionization potential of the neutral gas atoms. In this case, ionizing collisions of electrons with

<sup>(</sup>Received for publication 5 September 1973)

neutrals will occur, electron multiplication will result when the ionization rate exceeds the loss rate, and progressively more electrons will be present to be accelerated by the electric field. In microwave breakdown, which is the sudden onset of a high-frequency discharge, the ambient electron density is generally low enough for the electron loss to be by diffusion. Where production equals loss, the breakdown condition can be stated as an electron continuity equation: 1

$$\frac{\partial \mathbf{n}}{\partial t} = \mathbf{n} \mathbf{v}_{i} - \overset{\rightarrow}{\nabla} \cdot \overset{\rightarrow}{\Gamma} = \mathbf{0} , \qquad (1)$$

where  $u_i$  is the ionization frequency and  $\vec{\Gamma}$  is the electron flux to the walls. If the diffusion coefficient D is defined by setting the flux equal to the gradient of a diffusion potential, as

$$\vec{\Gamma} = -\vec{\nabla} (nD), \qquad (2)$$

then the continuity equation can be rewritten as

$$\frac{\partial n}{\partial t} = n \nu_i + \nabla^2 (nD) = 0. \tag{3}$$

This equation is separable, and eigenvalues can be found for the separation constant by considering boundary conditions. It is convenient to denote these eigenvalue solutions by  $\Lambda$ , a characteristic diffusion length whose value for various geometries is determined by the dimensions of the discharge walls. <sup>2</sup>

Using A yields the breakdown condition (analogous to the Fownsend criterion<sup>3</sup>):

$$\frac{\nu_i}{D} = \frac{1}{\Lambda^2} . \tag{4}$$

Next, the breakdown field is solved for by calculating  $v_i$  and D for various electric fields and iterating onto the electric field that satisfies (4).

# 1.2 The Steady-State Case

In a low-density, quiescent microwave discharge, electron diffusion losses are reduced by the space-charge fields set up by an excess residive charge of

<sup>1.</sup> Brown, S. C. (1966) Basic Data of Plasma Physics, M.I.T. Press, pp. 251, 252.

<sup>2.</sup> MacDonald, A.D. (1966) Microwave Breakdown in Gases, Wiley, pp. 27-29.

<sup>3.</sup> MacDonald, A.D. (1966) Microwave Breakdown in Gases, Wiley, pp. 69-72.

ions 'left behind' by their more rapidly diffusing electrons. Electron losses occurring in the transition from free electron diffusion to restricted ambipolar diffusion are described by a space-charge-controlled diffusion coefficient  $D_{\rm g}$ , defined in the steady state by  $^4$ 

$$D_{s} = \nu \Lambda^{2}. \tag{5}$$

The approximation to  $D_s$  given by Allis and  ${\rm Rose}^4$  is the constant-ratio approximation

$$D_{s} \approx D_{r} = r \left( \frac{\mu_{+}D_{-} + \mu_{-}D_{+}}{\mu_{+}r + \mu_{-}} \right),$$
 (6)

where r is the ratio of positive-ion concentration to electron concentration, and  $\mu_i$  is the mobility of species i.

In the space-charge-controlled case, where r=1, the usual artipolar-diffusion coefficient  $D_a$  is obtained by reducing  $D_r$ , as follows:

$$D_{\mathbf{r}} = \left(\frac{\mu_{+}D_{-} + \mu_{-}D_{+}}{\mu_{+} + \mu_{-}}\right) \equiv D_{\mathbf{a}}.$$

In the low-density free-diffusion case, where  $r = D_{/}D_{+}$ , the expression for  $D_{r}$  reduces to just  $D_{-}$ . In the intermediate range, a machine solution has shown that the approximation underestimates  $D_{r} < D_{s}$  by a factor of no more than 3. Manipulating the expression for  $D_{r}$  yields the more convenient form:

$$D_{\mathbf{r}} = D_{-} \left( \frac{1 + \frac{\mu_{-} D_{+}}{\mu_{+} D_{-}}}{1 + \frac{\mu_{-}}{\mu_{+} r}} \right). \tag{7}$$

To program this requires a suitable function for r; it must approach D\_/D\_+ for  $\lambda_D^2 >\!\!> \Lambda^2$  and unity for  $\lambda_D^2 <\!\!< \Lambda^2$ , where  $\lambda_D$  is the Debye length. This is satisfied by the following, which is chosen for convenience.

<sup>4.</sup> Allis, W.P., and Rose, D.J. (1954) The transition from free to ambipolar diffusion, Phys. Rev. 93(No. 1):84-^3.

$$\mathbf{r} = \frac{D_{-}}{D_{+}} \left[ \frac{1 + \frac{D_{+}}{D_{-}} \left(\frac{\Lambda}{\lambda_{\mathrm{D}}}\right)^{2}}{1 + \left(\frac{\Lambda}{\lambda_{\mathrm{D}}}\right)^{2}} \right]. \tag{8}$$

In Figure 1, where r is plotted as a function of  $(\hbar/\lambda_{D})^2$  it is observed to be proportional to  $(\hbar/\lambda_{D})^2$  in the transition, which is proportional to the electron density.<sup>4</sup> Substituting Eq. (8) into (7) for  $D_r$  yields

$$D_{r} = D_{-} \left[ \frac{1 + \frac{D_{+}}{D_{-}} \left(\frac{\Lambda}{\Lambda_{D}}\right)^{2}}{\frac{\mu_{+}D_{+} + \mu_{-}D_{+}}{\mu_{+}D_{-} + \mu_{-}D_{+}} \left(\frac{\Lambda}{\Lambda_{D}}\right)^{2}} \right].$$
 (9)

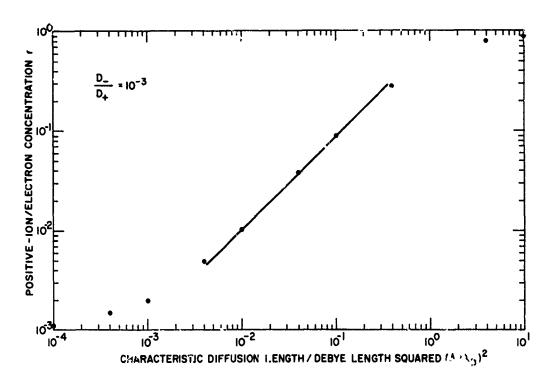


Figure 1. An Analytical Form for r is Necessary for Programing. Equation (8) has been chosen to model r and, as shown here, has the correct limits, as well as a linear region proportional to electron density.

This expression seems less awkward when reduced to a familiar form for a specific case. Consider the isothermal case, where  $D_{\perp}/\mu_{\perp}=D_{\perp}/\mu_{\perp}$ . Now

$$D_{r} = D_{-} \left[ \frac{1 + \frac{D_{+}}{D_{-}} \left(\frac{\Lambda}{\lambda_{D}}\right)^{2}}{1 + \frac{1}{2} \left(\frac{D_{+}}{D_{-}} + 1\right) \left(\frac{\Lambda}{\lambda_{D}}\right)^{2}} \right].$$
 (10)

If  $D_+/D_-$  in the denominator is neglected as small with respect to unity, then for the high-density case (where  $\lambda_D^2 \ll \Lambda^2$ ) the term  $D_r$  reduces to the ambipolar diffusion coefficient:

$$D_r = 2D_+ = D_{isothermal}$$
.

In this case of microwave breakdown, where the electron temperature far exceeds the ion temperature, such a restrictive isothermal approximation cannot be used. The ambipolar coefficient is then obtained by considering the high-density case in Eq. (9), from which

$$L_{\mathbf{r}} = \left(\frac{\mu_{+}D_{-} + \mu_{-}D_{+}}{\mu_{+} + \mu_{-}}\right). \tag{11}$$

This can be reduced by neglecting  $\mu_+$  as small with respect to  $\mu_-$ . Using the Einstein relation then yields

$$D_{r} = D_{+} \left( 1 + \frac{D_{-}/\mu_{-}}{D_{+}/\mu_{+}} \right) = D_{+} \left( 1 + \frac{T_{-}}{T_{+}} \right), \tag{12}$$

again, a familiar form.<sup>5</sup>

#### 1.3 The Preionization Case

As used here, the term <u>preionization</u> denotes the state of a gas that has been partially ionized by an energy source independent of, and consequently acting prior to application of, the microwave breakdown signal. It is assumed that the density of charged particles in the initial ionization that exists as an afterglow, for example, or as a weakly ionized discharge is sufficient to require consideration of charge-separation effects and subsequently a diffusion coefficient that is a function of electron density.

Oskam, H. J. (1958) Microwave investigation of disintegrating gaseous discharge plasmas, Philips Res. Rpts 13:367.

This breakdown criterion [Eq. (4) with a modified D] predicts that breakdown will occur for the preionization case, in the presence of an electric field lower than the field in the ambient case. The expression used for modeling the preionization case represents a constant-ratio approximation diffusion coefficient.

# 2. PROGRAM FUNCTIONS AND FUNCTION-TESTING

Bandel and MacDonald<sup>6</sup> have shown experimentally that the field required for microwave breakdown in neon is reduced by preionization. In my calculations of the breakdown fields in neon and neon-argon mixtures, I use a multiplicative factor to allow for the change in diffusion coefficient caused by preionization.

The breakdown field for neon has been calculated by MacDonald and Betts, who used a Boltzmann equation solution for the electron velocity distribution at both high and low pressures. The breakdown field for neon-argon mixtures has been similarly calculated by MacDonald and Matthews. 8

The computer program numerically calculates values for the distribution functions in the analytic expressions as they appear after integration over electron velocities, and yields the ionization and diffusion loss rates. For neon at low pressures, modified Bessel functions of imaginary arguments of the first and third kinds are required. For the other cases, the confluent hypergeometric functions are required. [The second solution given by MacDonald also requires the  $\Gamma$  and  $\psi$  functions for its evaluation.]

The modified Bessel functions were initially calculated from ordinary Bessel functions of the first and second kind by using imaginary arguments. It was later found that direct calculations using series and asymptotic representations are more efficient.

A simple program was written to test the functions. Each function subroutine was called for various values of its argument (ranging from  $10^{-4}$  to  $10^{2}$ ), and the value of the function was assigned to an array. After all the values of the argument had been computed, the arrays were primed out and checked against tables for the various functions. The exponential integral function and the modified Bessel function of the first kind were accurate to at least the eighth place. The

Bandel, H.W., and MacDonald, A.D. (1969) Effect of preionization on microwave breakdown in neon, J. Appl. Phys. 40(No. 11):4390-4394.

<sup>7.</sup> MacDonald, A.D., and Betts, D.D. (1952) High-frequency gas discharge breakdown in neon, <u>Canad. J. Phys.</u> 30:565-576.

<sup>8.</sup> MacDonald, A.D., and Matthews, J. H. (1955) High-frequency ionization coefficients in neon-argon mixtures, Phys. Rev. 98(No. 4):1070-1073.

modified Bessel function of the third kind was also accurate to eight places, except in the range of arguments around 15, where the algorithm changed from the series representation to the asymptotic form. Here the values were accurate to only four places. The confluent hypergeometric functions were checked against the limited range of values given by MacDonald<sup>9</sup> to the fourth place and were found to agree exactly. For values outside this range, the asymptotic representations were used. These agreed with NBS tables to the third significant figure, but it was found that the arguments of the functions used in the breakdown field programs were never in the asymptotic range except possibly in the first few iterations for a particular case.

#### 3. BREAKDOWN CALCULATIONS

#### 3.1 'False Position' Iteration

The field required to initiate breakdown was calculated by using a version of the 'false position' iteration technique. <sup>10</sup> In this technique, two initial values of the argument—in this case, the electric field—are assumed. They are then tested to find out whether the solution lies between them. If it does, a difference equation calculates a 'closer'guess, which is in turn tested, and so on. The method hinges on writing a function that has a solution when the function is equal to zero. The product of the function evaluated at the two trial (false position) values is negative if the solution lies between these two values, positive if the solution lies outside the bounds of these trial values.

Recall our breakdown criterion:

$$\frac{\nu_{\hat{\mathbf{I}}}}{\mathbf{D}} = \frac{1}{\Lambda^2} . \tag{4}$$

This equation can be rearranged into such a function, as follows:

$$f(x) = \frac{\nu_i}{D_r} - \frac{1}{\Lambda^2}. \tag{13}$$

A specific f(x) was written for each of the four cases: for neon, at high and low pressures; and for neon-argon, at high and low pressures. The iteration program was the same for all four cases.

<sup>9.</sup> MacDonald, A.D. (1949) Properties of the confluent hypergeometric function, J. Math. Phys. 28:183-191.

<sup>10.</sup> Conte, S.D. (1965) Elementary Numerical Analysis, McGraw-Hill, pp. 40-41.

The value of the gas pressure was incremented after each value of the electric field had been iterated onto for the various values of electron density at that gas pressure. After the electric field had been calculated for breakdown without preionization (in the program,  $n=1.0\,\mathrm{cm}^{-3}$ ), suitable values of electron density were used to span the region of transition from free diffusion to ambipolar diffusion.

Returning to Eq. (9) and setting

$$A = \frac{D_{+}}{D_{-}} \left( \Lambda^{2} e^{2} / \epsilon_{0} kT_{-} \right)$$
 (14)

and

$$B = \frac{\mu_{+}D_{+} + \mu_{-}D_{+}}{\mu_{+}D_{-} + \mu_{-}D_{+}} \left( \Lambda^{2} e^{2} / \epsilon_{0} kT_{-} \right)$$
 (15)

we can rewrite (9) as:

$$D_r = D_{-}\left(\frac{1+nA}{1+nB}\right). \tag{16}$$

The magnitude of factors A and B determines the electron density at which transition occurs. The ratio A/B represents the reduction in diffusion by space charge as expression (16) approaches A/B for large n. The ratio of A to B can be estimated from Eqs. (14) and (15), which yield:

$$\frac{D_{a}}{D_{-}} = \frac{A}{B} = \frac{\mu_{+} + \mu_{-}}{\mu_{+} + \mu_{-}}.$$
 (17)

Neglecting  $\mu_{+}$  as small with respect to  $\mu_{-}$  leads to

$$\frac{A}{B} = \frac{\mu_{+}}{\mu_{-}} + \frac{D_{+}}{D_{-}} . \tag{18}$$

Since the ratio of the diffusion coefficient to the mobility is equal to the average energy, then

$$\frac{\mu_{+}/\mu_{-}}{D_{+}/D_{-}} = \frac{D_{-}/\mu_{-}}{D_{+}/\mu_{+}} = \frac{\langle u_{-} \rangle}{\langle u_{+} \rangle}. \tag{19}$$

In the laboratory case of microwave breakdown, the electron average energy is much greater than the ion average energy. Thus, neglecting  $D_{\pm}/D_{\pm}$  as small compared with  $\mu_{\pm}/\mu_{\pm}$  leads to:

$$\frac{A}{B} = \frac{\mu_+}{\mu} . \tag{20}$$

In the Maxwellian equilibrium case, using the Einstein relation gives the ratio of electron-to-ion temperatures rather than the average energies of (19).

For purp ses of the program described here, the electron mobility was estimated by the following linear approximation to the data of Anderson  $^{11}$ 

$$v_{z} = 10^{6} E/p,$$

which yielded

$$\mu_{-} = \frac{v_{-}}{E} = \frac{10^{6}}{p} \frac{cm^{2}}{V-sec}$$
 (21)

For the ion mobility, the value  $4\,\mathrm{cm}^2/\mathrm{V}$ -sec was selected since experimental results  $^{12}$  give values for small E/p slightly larger than 4, decreasing with E/p. The term A was estimated by assuming the following:

$$\frac{D_+}{D_-} = \frac{\mu_+ T_+}{\mu_- T_-} ,$$

with

$$\mu_{+} = \frac{4 \text{ cm}^2}{\text{V-sec}}, \quad T_{+} = 300 \text{°K};$$

$$\mu_{-} = \frac{10^6}{p} \frac{cm^2}{V-sec}$$
,  $T_{-} = 3000$ °K.

Including the constants for neon yielded

$$A = p\Lambda^2(2.8 \times 10^{-12}).$$

<sup>11.</sup> Anderson, J. M. (1964) Hall effect and electron drift, Phys. Fluids 7:1517.

<sup>12.</sup> Chanin, L. M., and Biondi, M. A. (1957) Temperature-dependence of ion mobilities in helium, neon, and argon, Phys. Rev. 106:473-479.

Based on (20), the term B was estimated as:

$$B = \frac{A(2.5 \times 10^5)}{p}.$$

The ion mobility for the neon-argon mixture was assigned a value of 1.5 cm<sup>2</sup>/V-sec, on the assumption that practically all the ions were argon. Since electrons colliding with neutrals would still encounter neon neutrals to the extent of the concentration of neon (99 percent), the electron mobility of neon was used. Thus,

A = 
$$ph^{2}(1.12 \times 10^{-12})$$
,  
B =  $\frac{A(6.25 \times 10^{5})}{p}$ .

# 3.2 The Electron Energy Distribution

The Boltzmann equation solutions for the distribution function given by MacDonald and Betts<sup>7</sup> for neon, and by MacDonald and Matthews<sup>8</sup> for a neonargon mixture, were used to calculate the ionization rate and diffusion coefficient for particular values of an applied electric field. These calculations were then used in the breakdown criterion [Eq. (13) with (16) substituted] to iterate onto a breakdown field.

The Boltzmann equation is solved by assuming that the mean free path is constant for electrons of all energies and that all electron losses are by diffusion to the walls of the container. The characteristic diffusion length  $\Lambda$  for a cylinder of radius R and length L is found by using:

$$\frac{1}{\Lambda^2} = \left(\frac{\pi}{L}\right)^2 + \left(\frac{2.405}{R}\right)^2. \tag{22}$$

A spherical harmonic expansion of the distribution in velocity and a Fourier series expansion in time leads to the following energy balance equation:

$$-(h_{x}+h_{i}) f + \frac{2m}{Mu} \frac{d}{du}(u^{2}f) = \frac{\ell^{2}}{3\Lambda^{2}} f - \frac{E^{2}\ell^{2}}{3u} \frac{d}{du} \frac{u\nu^{2}}{\nu^{2}+\omega^{2}} \frac{df}{du}, \qquad (23)$$

where

 $\mathbf{h}_{\mathbf{X}}$  is the efficiency of excitation,  $\mathbf{h}_{\mathbf{i}}$  is the efficiency of ionization,

f is the electron velocity distribution function,

m is the electron mass,

M is the ion mass.

u is mv<sup>2</sup>/2e, in units of voltage,

¿ is the electron mean free path,

E is the rms amplitude of the microwave field,

ω is the angular frequency of the incident microwave field.

v is the momentum transfer collision frequency.

This equation is then solved by considering two cases—low pressure:  $v^2 \ll \omega^2$ ; and high pressure:  $\omega^2 \ll v^2$ . For each case, an expression is derived for f: (1) for energies below the excitation energy  $u_x$ , ignoring inelastic collisions, and (2) for energies above  $u_x$ , considering inelastic collisions. These expressions and their derivatives are then matched at  $u = u_x$  to give the constants in the solutions.

Next, the distribution function expressions are used to calculate the ionization rate for neon by

$$n\nu_{i} = 3\pi \frac{e^{2}}{m^{2}} \int_{u_{i}}^{\infty} \frac{u}{\ell} h_{i} f du \qquad (24)$$

and for the Penning mixture of neon-argon by

$$n \nu_i = 16\pi \left(\frac{e}{m}\right)^2 E^2 \left(\frac{t}{3}\right) \left(\frac{u}{2} \frac{df}{du}\right)_{u=u_i}.$$
 (25)

For both neon and neon-argon, the diffusion coefficient is calculated by

$$nD = \frac{8\pi t e^2}{3m^2} \int_0^{\infty} u f du.$$
 (26)

The ratio of  $n\nu_i$  and nD is then used in Eq.(13) with Eq.(16) providing a multiplicative factor to represent the charge-separation effect. This gives:

$$FCN = \frac{n\nu_i}{nD_r} - \frac{1}{\Lambda^2} = \frac{n\nu_i}{nD_r(TFA)} - \frac{1}{\Lambda^2},$$
 (27)

where FCN is the function f(x) of Eq. (13), and TFA is the multiplicative factor representing the transition from free to ambipolar diffusion.

These calculations were separated into four cases and, as previously noted (Sec. 3.1), a program written for each (neon, low pressure: neon, high pressure; neon-argon, low pressure; neon-argon, high pressure).

# 3.3 Comments

The iteration technique used in this work tests for convergence by comparing each new value of the electric field with the value from the preceding iteration. The value of the electric field is assumed to have converged to at least one significant figure less than the difference between two successive iterations. For example, if two successive iterations show a difference of no more than one-tenth in the value of the electric field, the result is assumed to have converged in the units place.

A false indication of convergence may result if the initial false positions, or bounds, for the value of the electric field are too far apart. The function  $f(\mathbf{x})$  is therefore printed out. If the printed-out value, which should be zero at the solution, is not small enough (approaching zero), the initial guesses (false positions) are edited.

Values of the arguments for the confluent hypergeometric functions are listed to see the range of arguments used and determine whether or not the asymptotic series is being used. These are listed separately from the breakdown results to facilitate checking values of  $f(\mathbf{x})$  for convergence and analyzing the breakdown results.

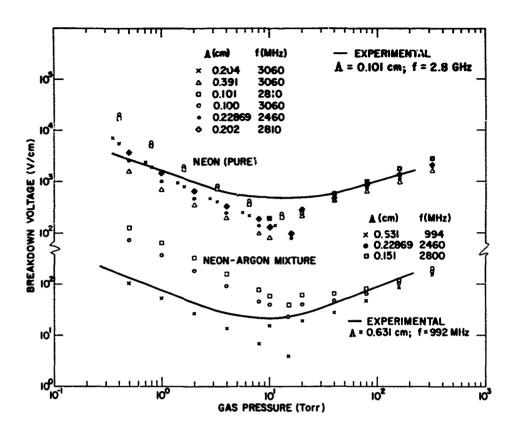
The values of the characteristic diffusion length and microwave frequency that were chosen were those of my experimental setup and the published experimental results of MacDonald. The results for the case without preionization are presented in Figure 2, together with the experimental results cited. Figures 3 to 5 present the results for the case with preionization, where the diffusion loss rate is modified by the function described in Sec. 1.2, and the reduction in breakdown field is seen as a function of electron density.

The program was written with an interactive feature, as follows. At the beginning of the program a line was printed at the teletype terminal requesting a value of characteristic diffusion length, followed by a line requesting a value of frequency. Consequently, any other value of these parameters could readily be entered through this procedure. For convenience, the ratio between the breakdown value of the elect c field with a given level of preionization and that without preionization was also calculated and printed out.

# 4. DISCUSSION

Brown<sup>13</sup> has briefly summarized the subject of high-frequency breakdown (Chap. 10). In describing the steady-state microwave discharge (Chap. 14) he

<sup>13.</sup> Brown, S.C. (1966) Introduction to Electrical Discharges in Gases, Wiley, Chap. 10 and p. 228.



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Figure 2. Experimental and Calculated Breakdown Field as a Function of Gas Pressure

presents experimental curves for hydrogen (p. 228) that show the balance of electron losses and gains in the discharge is maintained by a lower electric field than that required to initiate the discharge (see Figure 6). The difference is due to ambipolar diffusion in the sustaining case as contrasted with free diffusion for breakdown without preionization. When a microwave signal is propagating in the partially ionized gas of a reentry plasma sheath, therefore, breakdown can be expected at a lower field strength than in the case of an un-ionized signal path. 'Breakdown' is understood to mean that the electron density s increased to the critical density.

Among laboratory experiments showing that breakdown fields are reduced with preionization are those reported by Bethke and Reuss. 14 These workers have observed that a transient microwave breakdown phenomenon initiated by a glow discharge occurs at field strengths much lower than the field strength

<sup>14.</sup> Bethke, G. W., and Ruess, A.D. (1969) Microwave-induced plasma shield propagation in rare gases, Phys. Fluids 12:822.

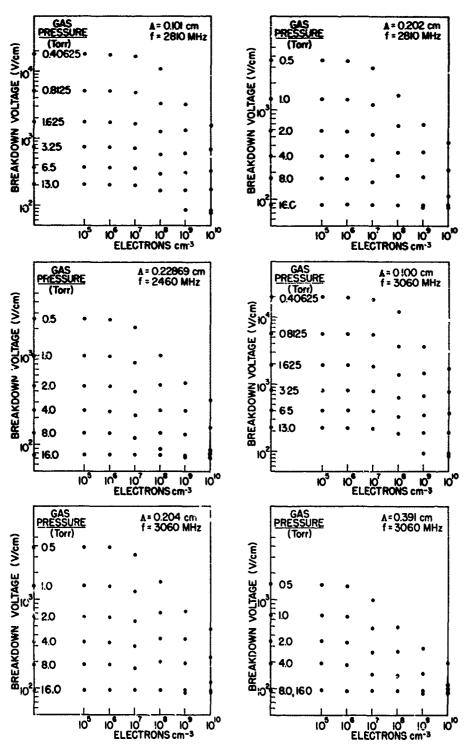


Figure 3. Breakdown Voltage Calculated as a Function of Preionization in Low-Pressure Neon

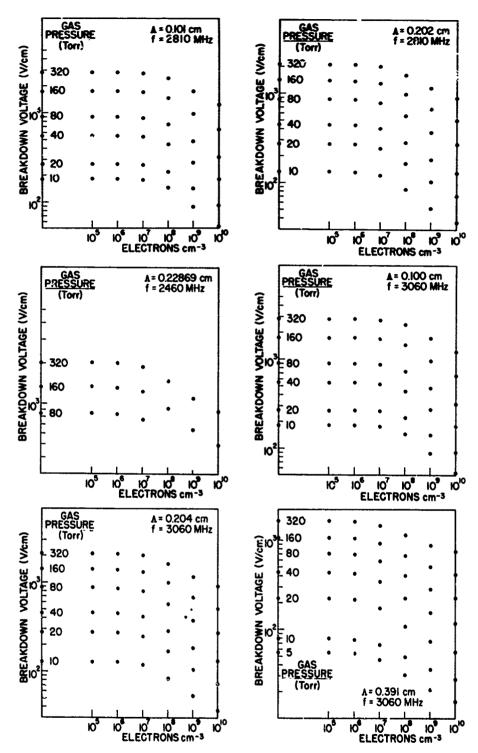


Figure 4. Breakdown Voltage Calculated as a Function of Preionization in High-Pressure Neon

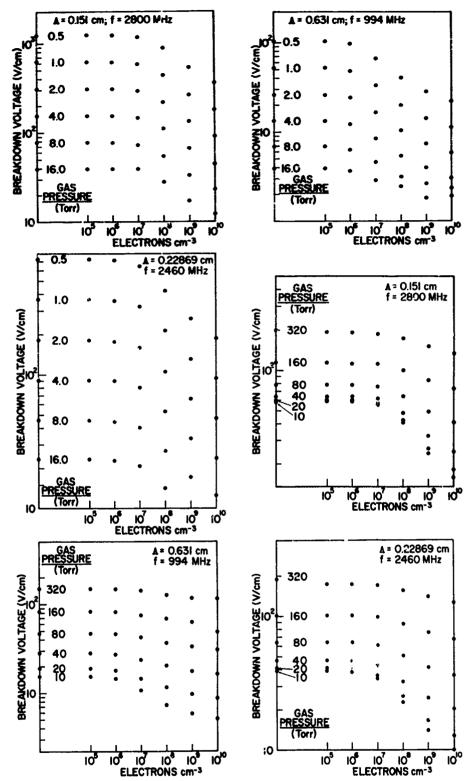


Figure 5. Breakdown Voltage Calculated as a Function of Preionization in Neon-Argon Mixtures

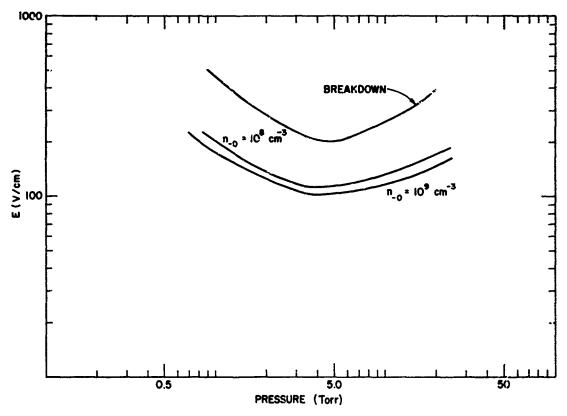


Figure 6. Maintenance Electric Fields for Steady-State Microwave Discharges Contrasted with Breakdown Values (Courtesy of S. C. Brown 13)

required to initiate breakdown without the glow discharge. The phenomenological interpretation presented for these results involves a complicated 'precursor' mechanism of resonance radiation that produces excited metastable and radiation-trapped states. It is postulated that these excited states may be ionized by electron impact with electrons oscillating in the microwave field and that at the level where the Debye length becomes comparable to the container dimensions, a charge-separation mechanism causes an electron avalanche to occur at this reduced breakdown threshold.

Bandel and MacDonald<sup>5</sup> have reported experimental measurements of the decrease in breakdown field strength in neon as a function of increased preionization that is the residual ionization from a previous breakdown pulse. A sufficient interpretation of their results is that the initial level of ionization determines a corresponding loss rate for electrons and that the breakdown field is the field whose ionization frequency will balance and infinitesimally exceed this loss rate. Since the population of metastable states is not known, no conclusions can be drawn as

to the mechanism of electron-impact ionization of metastables; however, the mechanism sennot be discounted as an alternative interpretation of the experiment without additional experimental data.

Recent studies <sup>15</sup> of a helium afterglow ir licate the plausibility of the ionizing metastable-metastable collision as a mechanism to account for discrepancies in experimental determinations of the electron-ion recombination coefficient. In a model for the afterglow that includes this mechanism as a source of free electrons, the electron density is directly proportional to the metastable density over a limited time interval in the afterglow. The correlation intuitively expected between density and microwave breakdown field is not by itself strong evidence in favor of any particular mechanism.

Experiments<sup>3</sup> have shown that microwave breakdown occurs at a lower electric field in neon-argon mixtures than it does in either of these gases in pure form. An interpretation is that the excitation of metastable states in the pure gas constitutes an energy sink that requires additional energy input to initiate breakdown. In a Penning mixture, this energy sink is destroyed by ionization of the 'impurity' atom. Similarly, in the experiment reported by Bethke and Ruess, <sup>14</sup> it may be hypothesized that the radiative precursor destroys this energy sink.

# 5. RESULTS

The breakdown fields calculated here show a reduction in the threshold voltage for breakdown as diffusion is modified by charge-separation effects. In these calculations the expressions involving the mobilities and diffusion coefficients for electrons and ions are replaced by constants A and B so that the transition is forced to occur at the same electron density. The pressure-dependence of the shape of the transition is taken care of by including the factor 1/p in the constant B. The results obtained in this program are compared with the experimental results of Bandel and MacDonald<sup>6</sup> in Figures 7(a) and (b). At both of the pressures used, it will be noted that the reduction in the breakdown field reaches a limit after about 10<sup>9</sup> electrons/cm<sup>3</sup>, thus showing good agreement between experiment and theory. This limit reflects the ratio of A/B in the calculation which is pressure-dependent through it in (20). In the 6.5-Tork case the calculated curve closely approximates the experimental curve. For the 0.35-Tork case, however, the experimental results show a sharper 'knee' between 10<sup>7</sup> and 10<sup>8</sup> electrons/cm<sup>3</sup> than the calculated values do. A somewhat higher electron temperature could be

Johnson, A.W., and Gerardo, J.B. (1972) Recombination and ionization in a molecular-ion-dominated helium afterglow, Phys. Rev. A5:1410.

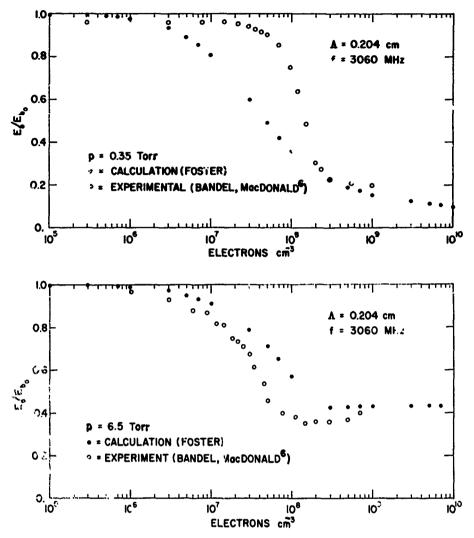


Figure 7. Preakdown Voltage Calculations Compared with Bandel and MicDonald's Experimental Results

expected at the lower pressure and hence, from (14), A and B would have smaller values as the electron temp, rature increased (the ratio of A to B remains unchanged by electron temperature). As a consequence, a higher value of electron density in (16) would be required to cause aB to become significant with respect to unity and hence the breakpoint would occur at higher electron unsities, and thus possibly more in agreement with experiment.

# 6. CONCLUSIONS

As a result of these calculations, we can consider the ambipolar diffusion mode as a consistent explanation of the mechanism producing experimental results such as those of Bandel and MacDonald; however, further experimental results are necessary to identify this mechanism as dominant in the preionization case. The ratio of breakdown fields for the un-ionized and the preionized cases is an observable that does not require accurate absolute measurement of the breakdown electric field or the preionization electron density, and is consequently emphasized here. An independent measurement of the actual metastable density in the preionization case is considered necessary to firmly establish the mechanism responsible for the phenomenon of Bethke and Ruess and the general problem of breakdown in the presence of preionization. Calculations are based on an assumed electron temperature that was selected as appropriate at moderate pressures. The assumption of a single value of electron temperature over three decades of pressure is, however, unjustifiable. Electron temperature variations-and if possible, as a function of afterglow time and electron density-need to be examined, especially since the constants A and B are roughly proportional to  $T_{-}^{2}$ , and the transition onset is thus very sensitive to electron temperature.

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